Time-Invariant Controllers for Target Tracking

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A method is described for designing time-invariant controllers to minimize root-mean-square target-tracking errors with root-mean-square control constraints in the presence of random initial conditions, target motions, and sensor errors. The maximum control responses usually occur during the initial transient period before the statistical steady state is reached. Hence we determine the controller parameters by minimizing a quadratic performance index that has *separate* weights on the transient and steady-state components of the control and tracking error. The designer makes tradeoffs between maximum control responses and steady-state tracking errors by varying the weights in this performance index. Two examples are given. The first is relatively simple to demonstrate the ideas, while the second is more complex, involving the recovery of a remotely piloted vehicle in a net on a rolling, pitching ship. The controller designs exhibit an improved performance over designs based on the minimization of the conventional performance index associated with linear quadratic Gaussian and parameter optimization methods.

Introduction

SEVERAL methods have been proposed to design target-tracking controllers. These include linear quadratic Gaussian (LQG) formulations,[1] which result in time-invariant full-order feedback/feedforward compensators, and parameter optimization methods,[2-7] which produce either full- or reduced-order compensator designs.

The LQG approach is based upon the minimization of a quadratic performance index (QPI) of the form (for the single-input/single-output case)

$$J_\infty = \lim_{t\to\infty} E\left[q e_\infty^2(t) + r u_\infty^2(t) \right] \tag{1}$$

$e_\infty(t), u_\infty(t) \equiv$ tracking error, control responses to random process and measurement noise

The QPI Eq. (1) provides a good measure of root-mean-square (rms) steady-state tracking accuracy and control effort and allows the direct determination of controller and estimator gains based on the solutions of matrix Ricatti equations.[8] However, Eq. (1) does not reflect the transient response of the controller to initial conditions. Hence, the minimization of Eq. (1) may produce tracking controller designs exhibiting good steady-state performance but with large transient control responses.

Parameter optimization methods are not limited to any particular form for the performance index as long as it can be expressed in terms of the compensator parameters. However, time-invariant controller design algorithms based on these methods (e.g., Refs. 5 and 6) have conventionally been structured assuming a QPI of the form

$$J = J_\infty + J_t \tag{2}$$

$$J_t = \lim_{t_f\to\infty} E\left[\int_0^{t_f} q e_t^2 + r u_t^2 \,\mathrm{d}t\right] \tag{3}$$

$e_t(t), u_t(t) \equiv$ tracking error, control responses to random initial conditions

where J_∞ is defined by Eq. (1) and J_t represents the cost due to initial condition transients. While the QPI Eq. (2) reflects both transient and steady-state criteria, the flexibility for trading off these criteria is limited by the assumption that the relative weighting ("q/r") between tracking accuracy and control effort is the same for both steady-state and transient cost terms J_∞ and J_t. In particular, the possibility of minimizing steady-state tracking error with a constraint on transient control amplitude is precluded, since there we desire $q=0$, $r\neq 0$ in J_t but $q\neq 0$, $r=0$ in J_∞.

We propose here a simple modification to the QPI Eq. (2) that significantly enhances the ability to design time-invariant tracking controllers exhibiting desirable steady-state and transient response characteristics. An extended version of an efficient parameter optimization algorithm developed by Ly[6] is proposed for the minimization of the modified QPI.

A Quadratic Performance Index for Use in Designing Time-Invariant Tracking Controllers

We are concerned with both the transient and the steady-state characteristics of the tracking performance. Typically, these characteristics are expressed in terms of the following criteria:

1) rms steady-state error
2) maximum rms transient error
3) settling time for rms transient error
4) rms steady-state control effort
5) maximum rms transient control effort
6) settling time for rms transient control effort (4)

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These criteria, and the tractability of minimizing integral-square performance indices for time-invariant linear systems, suggest determining the compensator parameters to minimize

$$J^* = \lim_{t_f \to \infty} E\left[\int_0^{t_f} q_t e_t^2 + r_t u_t^2 \, \mathrm{d}t\right] + E\left[q_\infty e_\infty^2 + r_\infty u_\infty^2\right]_{t\to\infty} \tag{5}$$

This quadratic performance index (QPI) differs from the QPIs used in previous approaches in that *independent* weighting factors q_t, r_t, q_∞, and r_∞ are now applied to the steady-state and transient tracking error and control terms. As demonstrated by design examples in a later section of this paper, this simple modification can result in tracking controller designs that can meet significantly tighter requirements imposed on the criteria in Eq. (4). The general form of the modified QPI Eq. (5) is next developed and expressed in terms of the compensator parameters.

Tracking Problem Formulation

We consider a time-invariant model of the system dynamics linearized about an equilibrium (reference) condition:

Plant / Target / Disturbance Dynamics:

$$\begin{bmatrix} \dot{x}_p \\ \dot{x}_T \end{bmatrix} = \begin{bmatrix} F_p & 0 \\ 0 & F_T \end{bmatrix}\begin{bmatrix} x_p \\ x_T \end{bmatrix} + \begin{bmatrix} G_p \\ 0 \end{bmatrix} u_p + \begin{bmatrix} \Gamma_p \\ \Gamma_t \end{bmatrix} w_r \tag{6}$$

$$y_c = \begin{bmatrix} H_{c_P} & H_{c_T} \end{bmatrix}\begin{bmatrix} x_p \\ x_T \end{bmatrix} + D_{cu} u_p \tag{7}$$

where

$x_p \equiv np$-vector of plant states

$x_T \equiv nt$-vector of target states

$u_p \equiv nc$-vector of plant controls

$w_r \equiv nd$-vector of random process/measurement errors

$y_c \equiv no$-vector of output criterion variables

Assumed Controller Structure:

$$\begin{aligned} u_p &= Cz + Dy_s \\ \dot{z} &= Az + By_s \end{aligned} \qquad K \equiv \left[\begin{array}{c|c} D & C \\ \hline B & A \end{array}\right] \tag{8}$$

$$y_s = \begin{bmatrix} H_{s_p} & H_{s_T} \end{bmatrix}\begin{bmatrix} x_p \\ x_T \end{bmatrix} + D_{su} u_p + D_{sw} w_r$$

where

$z \equiv nz$-vector of compensator states

$y_s \equiv nm$-vector of sensed (measured) variables

Closed-Loop Responses: The closed-loop state responses may be expressed as superpositions of two components

$$x^*(t) = x_t^*(t) + x_\infty^*(t) \tag{9}$$

where

$x^* \equiv (np + nt + nz)$-vector $= [x_p', x_T', z']'$

$(\cdot)_t =$ transient component responding to initial conditions

$(\cdot)_\infty =$ nondecaying component due to forcing by w_r

Dynamics of the individual components are given by

$$\begin{aligned} \dot{x}_t^* &= F^* x_t^* \\ \dot{x}_\infty^* &= F^* x_\infty^* + \Gamma^* w_r \end{aligned} \tag{10}$$

where, assuming $DD_{su} = 0$ for simplicity,

$$F^* = \begin{bmatrix} F_p + G_p DH_{s_p} & G_p DH_{s_T} & GC \\ 0 & F_T & 0 \\ B(I + D_{su}D)H_{s_p} & B(I + D_{su}D)H_{s_t} & A + BD_{su}D \end{bmatrix}$$

$$\Gamma^* \equiv \begin{bmatrix} GDD_{sw} + \Gamma_p \\ \Gamma_T \\ B(I + D_{su}D)D_{sw} \end{bmatrix} \tag{11}$$

Hence, the closed-loop output and control responses are of the forms

$$\begin{aligned} y_c &= y_{c_t} + y_{c_\infty} = H_{cc} x_t^* + H_{cc} x_\infty^* \\ u_p &= u_{p_t} + u_{p_\infty} = H_{cu} x_t^* + H_{cu} x_\infty^* \end{aligned} \tag{12}$$

where

$$H_{cu} = \begin{bmatrix} DH_{s_p} & DH_{s_T} & C \end{bmatrix}$$

$$H_{cc} = \begin{bmatrix} (H_{c_p} + D_{cu}DH_{s_p}) & (H_{c_T} + D_{cu}DH_{s_T}) & D_{cu}C \end{bmatrix} \tag{13}$$

Figure 1 illustrates output and control responses and their individual components for a typical case.

Random Initial Conditions:

$$E[x^*(0)] = [0] \qquad E[x^*(0)x^{*\prime}(0)] = X_0 \tag{14}$$

Random Process / Measurement Noise:

$$E[w_r(t)] = [0] \qquad E[w_r(t)w_r'(\tau)] = W_r\delta(t-\tau) \tag{15}$$

Quadratic Performance Index: Based upon the general definitions in Eq. (12) of the response components, we formulate a finite final time, quadratic performance index as

$$J^*(t_f) = J_t^*(t_f) + J_\infty^*(t_f) \tag{16}$$

where

$$\begin{aligned} J_t^*(t_f) &\equiv \text{transient cost} \\ &= E\left[\tfrac{1}{2}\int_0^{t_f} y_{c_t}' Q_t y_{c_t} + u_{p_t}' R_t u_{p_t} \, \mathrm{d}t\right] \end{aligned} \tag{17}$$

$$\begin{aligned} J_\infty^*(t_f) &\equiv \text{steady-state cost} \\ &= \tfrac{1}{2} E\left[y_{c_\infty}' Q_\infty y_{c_\infty} + u_{p_\infty}' R_\infty u_{p_\infty}\right]_{t=t_f} \end{aligned} \tag{18}$$

Ly[6] has shown that the use of a finite value of t_f in Eqs. (17) and (18) avoids computer overflow due to an infinite value of the performance index in cases where the initial guess for the free compensator parameters results in an unstable system. A parameter optimization algorithm is required to determine a local minimum of J^* with respect to the compensator parameters. This in turn requires that J^* be expressed analytically in terms of K. Combining Eqs. (12), (17), and (18), J^* can be expressed as

$$J^* = \tfrac{1}{2}Tr\left[(H_{cc}' Q_t H_{cc} + H_{cc}' R_t H_{cu}) X_t(t_f)\right] + \tfrac{1}{2}Tr\left[(H_{cc}' Q_\infty H_{cc} + H_{cu}' R_\infty H_{cu}) X_\infty(t_f)\right] \tag{19}$$

where

$$X_t(t_f) = \int_0^{t_f} \exp(F^*\tau)\, X_0 \exp(F^{*\prime}\tau)\, \mathrm{d}\tau$$

$$X(t_f) = \exp(F^*\tau)\Gamma^* W_r \Gamma^{*\prime} \exp(F^{*\prime}\tau)\, \mathrm{d}\tau \qquad (20)$$

Analytic expressions for the finite time integrals of Eq. (20) are derived in Ref. 6. In addition, analytic expressions for the gradients of J^* with respect to the compensator parameters (i.e., $\partial J^*/\partial K$), if available, can be used in the search algorithm to speed convergence to the solution for a local minimum. Accordingly, an expression for $\partial J^*/\partial K$ has been derived and is reported in Ref. 9.

Formulation for Improved Robustness

The modified QPI Eq. (16) is readily adapted to the method proposed by Ly and Cannon[2] for the design of controllers exhibiting improved robustness to system parameter uncertainties that can be described by a finite number of linearized state models. In this case, the performance index becomes

$$J_R^* = \sum_{i=1}^{np} \rho^i J^{*i}$$

where J^{*i} is the modified performance index Eq. (16) evaluated for the state dynamics model corresponding to the ith set of system parameter values, ρ^i is an arbitrary scalar weighting factor, and np is the total number of state models used to reflect parameter uncertainties. The reader is referred to Ref. 9 for further discussion and examples illustrating design in the presence of parameter uncertainties using J_R^*.

Design Method

A method for the selection of weights in the modified QPI is now described for the single-control single-performance variable case (an analogous strategy applies to the multivariable case—see Ref. 9):

1) Define "quadratic surrogate" terms to represent those criteria of Eq. (4) of interest for the given problem:

$$y_{\infty M}^2 = \frac{E\left[y_\infty^2(t_f)\right]}{y_{\infty \mathrm{MAX}}^2} \qquad \text{for criterion 1}$$

$$y_{t_M}^2 = \frac{E\left[\int_0^{t_f} y_t^2\, \mathrm{d}t\right]}{y_{t\mathrm{MAX}}^2 t_{sy}} \qquad \text{for criteria 2 and /or 3}$$

$$u_{\infty M}^2 = \frac{E\left[u_\infty^2(t_f)\right]}{u_{\infty \mathrm{MAX}}^2} \qquad \text{for criterion 4}$$

$$u_{t_M}^2 = \frac{E\left[\int_0^{t_f} u_t^2\, \mathrm{d}t\right]}{u_{t\mathrm{MAX}}^2 t_{su}} \qquad \text{for criteria 5 and/or 6}$$

where

$(\cdot)_{\mathrm{MAX}}$ = maximum allowable (constraint) or desired (goal) expected value of $(\cdot)$

t_{sy} = maximum allowable (constraint) or desired (goal) settling time for rms transient response of performance variable

t_{su} = maximum allowable (constraint) or desired (goal) settling time for rms transient response of control variable

Specify the initial values for $(\cdot)_{\mathrm{MAX}}$, t_{sy}, and t_{su}.

2) Formulate the modified QPI using the quadratic surrogate terms, grouping together those terms for which inequality constraints (i.e., numerical bounds) are specified and placing a scalar weighting factor ρ on this group, e.g.,

$$J^* = y_{\infty M}^2 + y_{t_M}^2 + \rho\left(u_{\infty M}^2 + u_{t_M}^2\right)$$

3) Determine the compensator parameter values that minimize J^* (e.g., see the following computer code). If the resulting solution satisfies all constraints, decrease ρ. If one or more constraints are violated, increase ρ and determine the new minimizing solution. Repeat this procedure until all inequality constraints are satisfied with one or more being met in the "equality" sense.

This strategy will in many cases yield a design solution exhibiting very satisfactory behavior for the unconstrained performance criteria. In some cases, however, it will be necessary to modify the weighting factors in the individual quadratic surrogate terms in 1 and repeating 2 and 3 in a trial and error fashion until an acceptable design is obtained.

Computer Code

The modified performance index J^* has been incorporated into a revised version ("SANDY1") of the FORTRAN computer code "SANDY" developed by Ly for the synthesis of low-order, robust controllers using a parameter optimization approach (see Refs. 6 and 9). SANDY1 allows the designer to produce significantly improved controllers for target-tracking in the presence of random target motions, random sensor noise, and random initial conditions. The improved designs are realized by the minimization of the modified QPI Eq. (16), which allows for separate weighting on transient and steady-state criteria so that design objectives are more accurately reflected.

Design Example 1

Second-Order Plant Tracking a Randomly Forced Second-Order Target

We wish to design a controller based on feedback of the tracking error so that the position y_p of a double-integrator plant tracks the position y_T of a second-order stable target subject to random disturbances. The initial rms value of the plant position is 25 ft, while the rms value of the target position/velocity has reached a statistically steady state of 1 ft/2 fps. Units are in feet and seconds.

Plant:

$$\begin{bmatrix} \dot{y}_p \\ \dot{v}_p \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} y_p \\ v_p \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_p + \begin{bmatrix} 0 \\ 1 \end{bmatrix} w_p$$

$$X_{p0} = \begin{bmatrix} 625 & 0 \\ 0 & 0 \end{bmatrix} \qquad (21)$$

$$E\left[w_p(t)\right] = 0$$

$$E\left[w_p(t) w_p(\tau)\right] = W_p \delta(t-\tau) \qquad W_p = 4$$

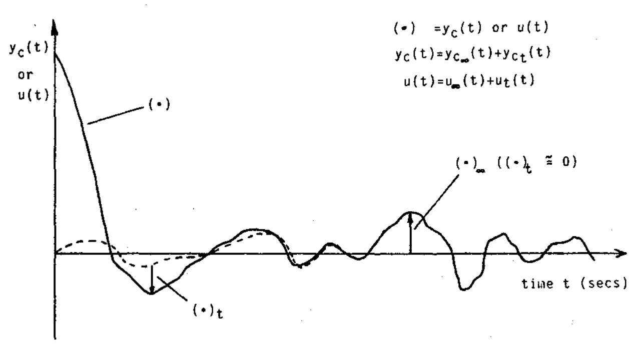

Fig. 1 Illustration of definition for transient $(\cdot)_t$ and nondecaying steady-state $(\cdot)_\infty$ tracking criteria used in the modified QPI Eq. (5).

Target:

$$\begin{bmatrix} \dot{y}_T \\ \dot{v}_T \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -4 & -2 \end{bmatrix} \begin{bmatrix} y_T \\ v_T \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} w_T$$

$$X_{T0} = \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix} \tag{22}$$

$$E[w_T(t)] = 0$$

$$E[w_T(t)w_T(\tau)] = W_T\delta(t-\tau) \qquad W_T = 16$$

Tracking Error:

$$e = y_T - y_p \tag{23}$$

X_{T0} is the steady-state covariance of the target states corresponding to the specified value of W_T. Hence, the rms target position amplitude is constant and equal to 1 ft for all $t > 0$.

Design Constraints:
1) rms steady-state error ≤ 0.5 ft
2) maximum rms transient error ≤ 25 ft
3) settling time of rms transient error ≤ 1 s to 2.5 ft (24a)

Design Goals:
1) rms steady-state control effort ≤ 1000 ft/s^2
2) maximum rms transient control effort ≤ 1000 ft/s
3) settling time of rms transient control effort ≤ 0.25 s (24b)

Assumed Control Structure (full-order feedback of noisy tracking error measurement):

$$u_p = [C]z$$

$$\dot{z} = [A]z + [B]y_s$$

$$y_s = e + v_s \qquad E[v_s(t)] = 0$$

$$E[v_s(t)v_s(\tau)] = 0.00002\delta(t-\tau) \tag{25}$$

A *minimal* realization form is assumed for the compensator matrices A, B, and C to eliminate the possibility of redundant parameters in the controller optimization process (see Ref. 3):

$$[C] = [0 \quad -1 \quad 0 \quad -1] \qquad [B] = [b_1 \quad b_2 \quad b_3 \quad b_4]'$$

$$[A] = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -a_{21} & -a_{22} & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -a_{43} & -a_{44} \end{bmatrix} \tag{26}$$

The free parameters of A, B, and C (i.e., a_{21}, a_{22}, a_{43}, a_{44}, b_1, b_2, b_3, and b_4) are to be determined by the minimization of the modified QPI specified next.

Design Approach

We have

$$e(t) = e_\infty(t) + e_t(t)$$

$$u_p(t) = u_{p\infty}(t) + u_{pt}(t) \tag{27}$$

We choose

$$J^* = E\left[\rho\left(\frac{e_\infty^2}{q_\infty}\bigg|_{t_f} + \int_0^{t_f}\frac{e_t^2}{q_t}\,\mathrm{d}t\right) + \frac{u_\infty^2}{r_\infty}\bigg|_{t_f} + \int_0^{t_f}\frac{u_t^2}{r_t}\,\mathrm{d}t\right] \tag{28}$$

where the weighting factors q_∞, q_t, r_∞, and r_t are determined from Eqs. (24) in accordance with the weight selection procedure described earlier:

$$q_\infty = e_{\infty\mathrm{MAX}}^2 = 0.25\ \mathrm{ft}^2$$

$$r_\infty = u_{\infty\mathrm{MAX}}^2 = 10^6\ \mathrm{ft}^2/\mathrm{s}^4$$

$$q_t = e_{t_{\mathrm{MAX}}}^2 t_{sy} = 625\ \mathrm{ft}^2/\mathrm{s}$$

$$r_t = u_{t_{\mathrm{MAX}}}^2 t_{su} = 250000\ \mathrm{ft}^2/\mathrm{s}^3 \tag{29}$$

The time interval t_f is chosen to be very large (5000 s) to ensure a stable minimizing solution. Zero weighting is given to steady-state control effort in anticipation that this will not be large enough to cause concern. The solution is obtained from the SANDY1 algorithm by increasing the scalar ρ until the error constraints are met (i.e., one constraint met in the equality sense, others in the inequality sense).

Design Evaluation

Table 1 summarizes the minimizing solution of Eq. (28) with that obtained from the LQG theory using the QPI

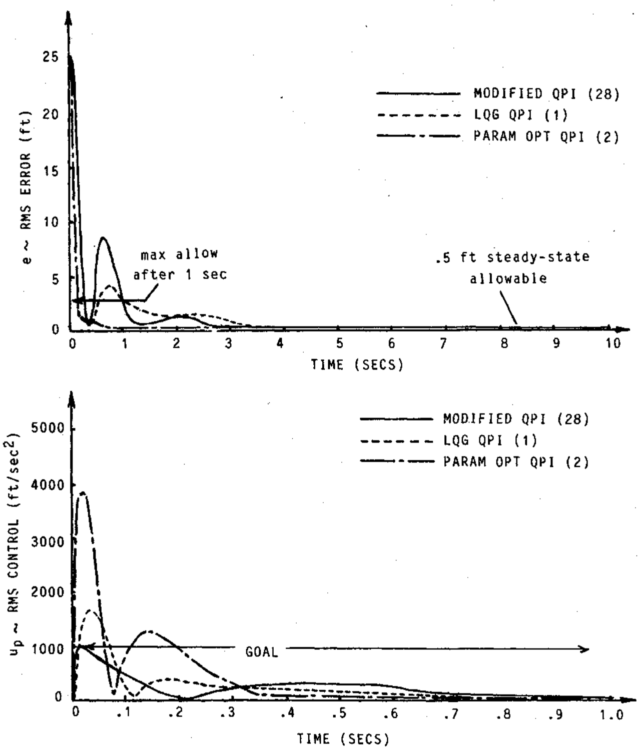

Fig. 2 RMS time history comparison for different QPI designs.

Table 1 Comparison of minimizing solutions for different QPIs

	LQG QPI, Eq. (1)	"Conventional" parameter optimization QPI, Eq. (2)	Modified QPI, Eq. (28)
QPI weighting factor value	$q/r = 6.4$	$q/r = 2000$	$\rho = 0.376$
Closed-loop plant eigenvalues, rad/s	$-22.3 \pm j22.4$	$-46.2, -28.1$	$-353.1, -13.7$
	$-2.52 \pm j2.52$	$-8.94 \pm j7.3$	$-5.0, -0.81$
	$-0.83 \pm j1.05$	$-13.1, -0.08$	$-2.33 \pm j4.75$
Closed-loop plant performance			
rms e_∞, ft	0.499	0.501	0.506
t_{set} to 0.6′, s	3.4	0.490	2.6
rms u_{p_∞}, fps^2	8.1	12.250	9.25
max rms u_p, fps^2	1660	3820	995

formulation Eq. (1) and with the solution obtained by minimizing the "conventional" QPI Eq. (2) using previous parameter optimization algorithms. In each of the latter two cases, the solution is obtained by setting $r = 1$ and varying q until the error constraints are met (one met in the equality sense). In all cases, the solutions were limited by the steady-state error constraint. The modified QPI solution satisfies the error criteria with the lowest peak control excursion, as evidenced by the rms error and control time histories of Fig. 2. The larger transient control excursions associated with the QPI Eqs. (1) and (2) are indicative of the inability to call for a high q/r for the steady-state performance and a low q/r for the transient performance.

Figure 3 displays rms steady-state error vs associated peak control excursion for controller designs obtained using the modified QPI Eq. (28) compared with designs obtained using the QPI Eqs. (1) and (2). It is clear that the use of Eq. (28) produces significantly improved tracking accuracy for the same control effort or, alternatively, the same tracking accuracy with less control effort than can designs obtained using Eqs. (1) or (2). The amount of improvement depends upon the specified design region of interest for the problem. For an rms steady-state error design goal of 0.5 ft, the maximum rms control excursion with QPI Eq. (28) is 40% less than with the LQG QPI Eq. (1) and 75% less than with the conventional parameter optimization QPI Eq. (2). Alternatively, for a maximum rms control excursion constraint of 1000 ft/s^2, the rms steady-state error with the modified QPI Eq. (28) is 38% less than with the LQG QPI Eq. (1) and 54% less than with the conventional parameter optimization QPI Eq. (2).

Design Example 2

Remotely Piloted Vehicle Net Capture by Ship

A study has been made recently[10] to determine the requirements for the shipboard recovery of remotely piloted reconnaissance vehicles (RPVs). In this example, we use the modified QPI formulation Eq. (16) to synthesize a low-order tracking autopilot for controlling RPV lateral motions during the final approach (see Fig. 4). The geometry associated with this example is shown in Fig. 5. Prior to the final approach, the RPV lateral responses are controlled by a heading-hold autopilot and have reached a statistically steady state due to lateral wind disturbances. At $t = t_0$, the switch to the tracking control mode occurs, resulting in transient error and control responses that gradually settle to a new steady-state condition forced by lateral winds, measurement noise, and ship motions at the target (center of net). For a closing velocity of 200 ft/s and an initial range of 10,000 ft, impact with the net will occur nominally at $t = t_0 + 50$ s.

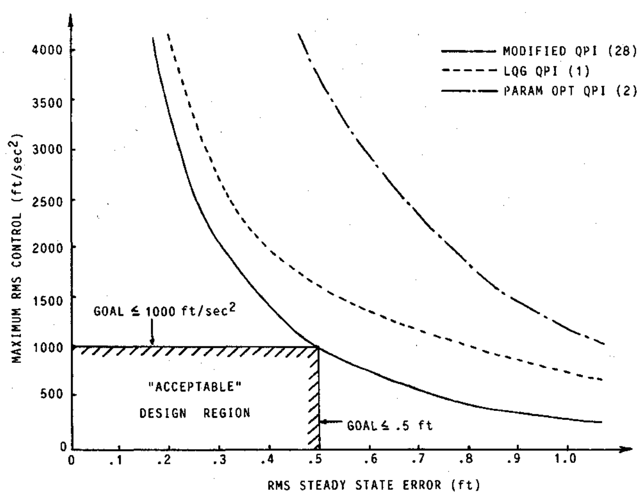

Fig. 3 Performance characteristics of different QPI designs.

Figure 6 depicts the proposed tracking control scheme. A "pseudomeasurement" of the tracking error signal e_y is generated onboard the RPV by electronically multiplying the line-of-sight angle measurement (provided by a gimballed seeker on the RPV) by an a priori estimate of the range-to-go. Onboard inertial instruments sense the RPV variables ψ (heading angle), r (heading angle rate), and ϕ (bank angle). Inertial instrumentation on the ship provides a measurement of target lateral position y_T that, if necessary, could be uplinked to the RPV autopilot to enhance performance.

We wish to design the tracking control system such that the following objectives are met:

1) rms steady-state lateral position error ≤ 3 ft
2) maximum rms transient position error ≤ 25 ft
3) maximum rms transient bank angle ≤ 20 deg
4) maximum rms transient lateral acceleration (body axes) ≤ 0.2 g
5) settling times for rms transient performance variables ≤ 30 s
6) rms steady-state aileron and rudder deflections ≤ 15 deg
7) maximum rms transient aileron and rudder deflections ≤ 15 deg
8) settling times for rms transient aileron and rudder deflections ≤ 1 s (30)

The control variable criteria Eqs. (6–8) are hard constraints that must be satisfied, while the remaining criteria are considered as desired goals for the control system performance.

The SANDY1 algorithm is used to design low-order tracking autopilots with and without target-state feedforward. The resulting designs are evaluated to determine whether an error-feedback-only control structure satisfies the objectives of Eq. (30) so that the feedforward uplink from the ship to the RPV would not be required. The lateral dynamics of this problem are described next.

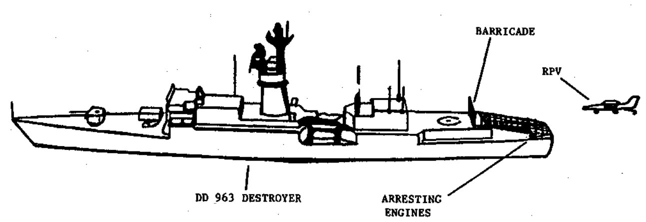

Fig. 4 Inclined arresting net recovery system for RPV.

Table 2 Comparison of optimized tracking autopilot designs using modified QPI

	NoFF	FF4	FF2
QPI weighting factor value ρ	1923	1100	1282
Closed-loop RPV eigenvalues, rad/s	$-103.2, -5.7$ $-34.6 \pm j50.9$ $-0.61 \pm j2.6$ $-0.22 \pm j1.2$ $-0.04 \pm j0.09$	$-37.7, -8.92$ $-5.03, -0.028$ $-0.77 \pm j2.9$ $-0.40 \pm j1.19$ $-0.48 \pm j0.26$	$-22.0, -0.04$ $-0.48 \pm j1.86$ $-0.13 \pm j0.99$ $-0.35 \pm j0.50$
Closed-loop performance			
rms e_∞, ft	3.34	2.73	2.72
t_{set} to 5′, s	38.8	5.0	7.5
max rms ϕ, deg	9.3	11.2	15.5
max rms a, g's	0.16	0.16	0.16
rms $(\delta_a, \delta_r)_\infty$, deg	(3.23, 2.7)	(2.46, 0.50)	(1.89, 0.05)
max rms $(\delta a, \delta_r)$, deg	(14.7, 3.1)	(15.2, 1.9)	(14.5, 0.44)

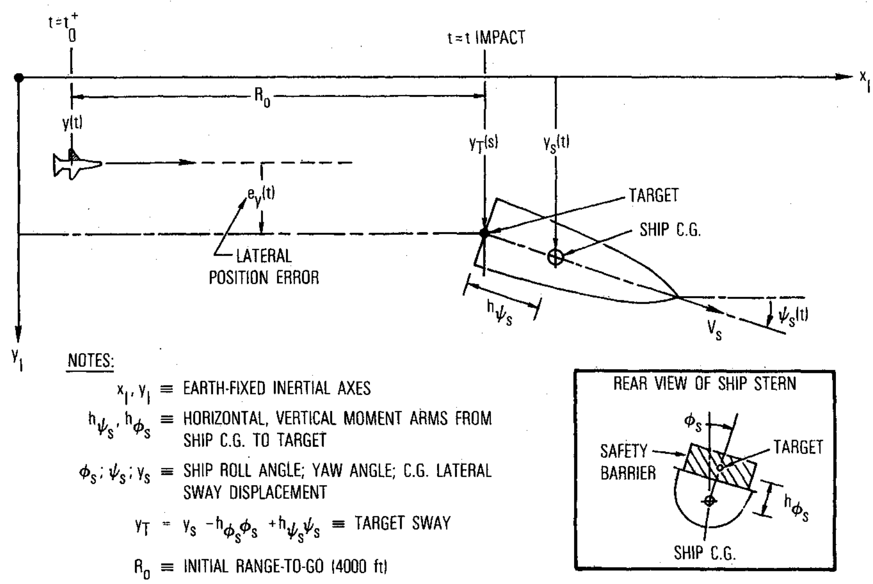

Fig. 5 Lateral plane geometry for RPV net capture example.

RPV Dynamics:

$$\begin{bmatrix}\dot v\\ \dot r\\ \dot p\\ \dot\phi\\ \dot\psi\\ \dot y\end{bmatrix}=\begin{bmatrix}-0.143 & -2.0 & 0 & 0.3214 & 0 & 0\\ 0.483 & -0.233 & 0.06 & 0 & 0 & 0\\ -3.94 & 0.289 & -1.35 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 0 & 0\\ 1 & 0 & 0 & 0 & 2.0 & 0\end{bmatrix}\begin{bmatrix}v\\ r\\ p\\ \phi\\ \psi\\ y\end{bmatrix}$$

$$+\begin{bmatrix}0.0075 & 0.13\\ -0.19 & -4.1\\ 3.79 & 3.27\\ 0 & 0\\ 0 & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}\delta_a\\ \delta_r\end{bmatrix}+\begin{bmatrix}0.143 & 0\\ -0.483 & -0.06\\ 3.94 & 1.35\\ 0 & 0\\ 0 & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}v_w\\ p_w\end{bmatrix} \quad (31)$$

where

$v \equiv$ sideslip velocity (ft/s)

$r \equiv$ heading angle rate (10^{-2} rad/s)

$p \equiv$ bank angle rate (10^{-2} rad/s)

$\phi \equiv$ bank angle (10^{-2} rad)

$\psi \equiv$ heading angle (10^{-2} rad)

$y \equiv$ lateral position (ft)

$\delta_a \equiv$ aileron deflection (10^{-2} rad)

$\delta_r \equiv$ rudder deflection (10^{-2} rad)

$v_w \equiv$ lateral wind component (ft/s)

$p_w \equiv$ roll wind component (10^{-2} rad/s)

Lateral Wind Disturbance Dynamics:

$$\begin{bmatrix}\dot v_w\\ \ddot v_w\\ \dot p_w\end{bmatrix}=\begin{bmatrix}0 & 1 & 0\\ -245 & -34.2 & 0\\ 0 & 0 & -15\end{bmatrix}\begin{bmatrix}v_w\\ \dot v_w\\ p_w\end{bmatrix}+\begin{bmatrix}0 & 0\\ 245 & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}n_v\\ n_p\end{bmatrix}$$

$$E\begin{bmatrix}n_v(t)n_v(\tau) & n_v(t)n_p(\tau)\\ n_p(t)n_v(\tau) & n_p(t)n_p(\tau)\end{bmatrix}=\begin{bmatrix}1.48 & 0\\ 0 & 0.013\end{bmatrix}\delta(t-\tau) \quad (32)$$

Table 3 Comparison of FF4 autopilot designs using modified vs conventional QPIs

Performance index	FF4 Modified ($\rho = 50$)	FF4 Conventional ($\rho = 0.61$)	FF4 Conventional ($\rho = 0.0006$)
Closed-loop RPV eigenvalues, rad/s	$-37.7, -8.92$	$-42.82, -0.184$	$-27.69, -4.60$
	$-5.03, -0.028$	$-7.46 \pm j10.51$	$-24.76 \pm j106.5$
	$-0.77 \pm j2.9$	$-2.12 \pm j2.42$	$-2.18 \pm j3.48$
	$-0.40 \pm j1.19$	$-0.20 \pm j1.20$	$-0.99 \pm j1.93$
	$-0.48 \pm j0.26$	$-0.43 \pm j0.40$	$-0.35 \pm j0.05$
Closed-loop performance			
rms e, ft	2.73	3.02	2.75
t_{set} to 5,	5.0	3.3	1.4
max rms, deg	11.2	16.0	81.0
max rms, g's	0.16	0.16	0.5
rms $(\delta_a, \delta_r)_\infty$ deg	(2.46, 0.50)	(2.12, 0.51)	(41.6, 26.7)
max rms (δ_a, δ_r), deg	(15.2, 1.9)	(15.5, 1.89)	(> 45.0, 34.0)

Target Dynamics: A second-order, stochastic model is used to approximate ship lateral motion at the target position (center of net) as reported by Fortenbaugh[11] for a DD963 destroyer in Sea State 5 ("rough seas") conditions:

$$\begin{bmatrix}\dot y_T\\ \dot v_T\end{bmatrix}=\begin{bmatrix}-0.23 & 0.5\\ -0.5 & -0.23\end{bmatrix}\begin{bmatrix}y_T\\ v_T\end{bmatrix}+\begin{bmatrix}2.897\\ -1.333\end{bmatrix}n_T$$

$$E[n_T(t)] = 0 \qquad E[n_T(t)n_T(\tau)] = 1.0\delta(t-\tau) \quad (33)$$

The resulting rms value of y_T, representing the overall sway motion at the target, is 3 ft.

Controller Structure: A general fourth-order control structure, calling for feedback of lateral position error, RPV heading angle, bank angle, heading angle rate, and feedforward of target lateral position, is modeled in the time domain as

$$[\delta_a, \delta_r]' = [C]z$$

$$\dot z = [A]z + [B]y_s$$

$$y_s' = [y_T - y, \psi, \phi, r, y_T]' + v_s' \quad (34)$$

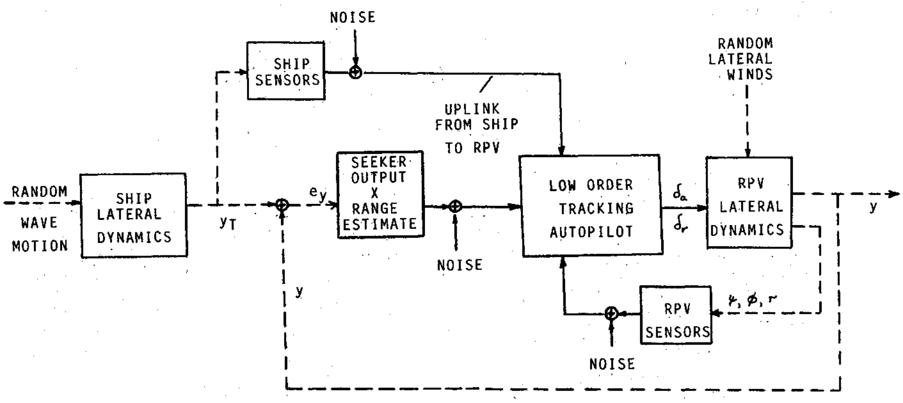

Fig. 6 General layout of RPV tracking control system.

where

$$E[v_s(t)] = [0] \qquad E[v_s(t)v_s'(\tau)] = R_s\delta(t-\tau)$$

$$R_s = \text{diag}[0.36 \quad 0.003 \quad 0.0122 \quad 0.00122 \quad 0.1225] \tag{35}$$

where a *minimal* realization form is assumed for the compensator matrices A, B, and C:

$$[C] = \begin{bmatrix} 0 & -1 & 0 & -1 \\ C_{21} & C_{22} & C_{23} & C_{24} \end{bmatrix}$$

$$[B] = \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} & b_{15} \\ b_{21} & b_{22} & b_{23} & b_{24} & b_{25} \\ b_{31} & b_{32} & b_{33} & b_{34} & b_{35} \\ b_{41} & b_{42} & b_{43} & b_{44} & b_{45} \end{bmatrix}$$

$$[A] = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -a_{21} & -a_{22} & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -a_{43} & -a_{44} \end{bmatrix} \tag{36}$$

Performance Index and Design Approach

We proposed the following performance index to reflect the design objectives of Eq. (30):

$$J = \rho E\left[\frac{e_\infty^2}{q_\infty}\bigg|_{t_f=50} + \int_0^{50}\left(\frac{e_t^2}{q_{e_t}} + \frac{a_t^2}{q_{a_t}} + \frac{\phi_t^2}{q_{\phi_t}}\right)\mathrm{d}t\right]$$
$$+ E\left[\frac{u_\infty^2}{r_\infty}\bigg|_{t_f=50} + \int_0^{50}\frac{u_t^2}{r_t}\,\mathrm{d}t\right] \tag{37}$$

where

$e \equiv$ lateral tracking error $= y_T - y$

$a \equiv$ RPV lateral acceleration $= -0.143v + 0.0075\delta_a + 0.13\delta_r$

$\phi \equiv$ RPV bank angle $= \phi$

$u^2 \equiv$ sum square control deflections $= \delta_a^2 + \delta_r^2$

The weighting factors q_{e_∞}, q_{e_t}, q_{a_t}, q_{ϕ_t}, r_∞, and r_t were initially chosen in accordance with the weight selection procedure described earlier. The resulting design solution exhibited an unacceptably high (> 5 m) steady-state error, though the transient error behavior was quite satisfactory. These preliminary results suggested increasing the relative weighting on the steady-state error term. After several iterations of trial and error, the final values settled upon for the weights were $q_{e_\infty} = 0.02$, $q_{e_t} = 1000$, $q_{a_t} = 3200$, $q_{\phi_t} = 17{,}500$, $r_\infty = 690$, and $r_t = 690$.

For each control structure to be optimized, the scalar weighting factor ρ on the control variable terms is increased until the design solution from SANDY1 results in the constraints of Eq. (30) on the steady-state and transient control responses being met (one met in the equality sense). The resulting optimized performance variable responses are then compared with the corresponding objectives in Eq. (30).

Design Evaluation

Three low-order tracking control structures were optimized by SANDY1 using the performance index Eq. (37): 1) fourth-order with no feedforward (NoFF), 2) fourth-order with target position feedforward (FF4), and 3) second-order with target position feedforward (FF2). The NoFF controller was obtained from the general structure of Eq. (36) by constraining all elements in the last two columns of the B matrix to be zero. Similarly, the FF2 structure is derived from Eq. (36) by deleting the appropriate rows and columns of C, B, and A.

RMS performance variable and control responses are compared in Fig. 7, and closed-loop system characteristics are provided in Table 2. In satisfying the control constraints, each optimized controller is successful in meeting the lateral acceleration and bank angle objectives. However, the controller without feedforward (NoFF) fails to meet the goals for transient error settling time and rms steady-state error. If target position feedforward is used with the fourth-order compensation (FF4), the rms steady-state error is reduced by 19%, the error settling time is reduced by 34 s relative to the NoFF controller, and all the design objectives of Eq. (30) are met. While the performance obtained with the lower order FF2 controller using feedforward is slightly degraded relative to the FF4 (2.5 s longer settling time and 40% higher maximum bank angle), all of the goals of Eq. (30) are still seen to be satisfied.

For the FF4 controller, Table 3 indicates the advantage of designing with the modified QPI Eq. (37) as opposed to a conventional performance index in which the transient and steady-state terms are constrained to be equally weighted. For the latter,

$$q_{e_\infty} = q_{e_t} = 1 \qquad q_{a_t} = q_{\phi_t} = 0$$
$$r_\infty = r_t = 1$$

in Eq. (37). If the scalar weighting ρ is increased until all control constraints are met (with the transient aileron deflection constraint met in the equality sense), it is seen that the conventional QPI produces a design exhibiting a 9% higher rms steady-state tracking error. While this happens to satisfy the stated design goal of 3.0 ft or less, consider a case in which

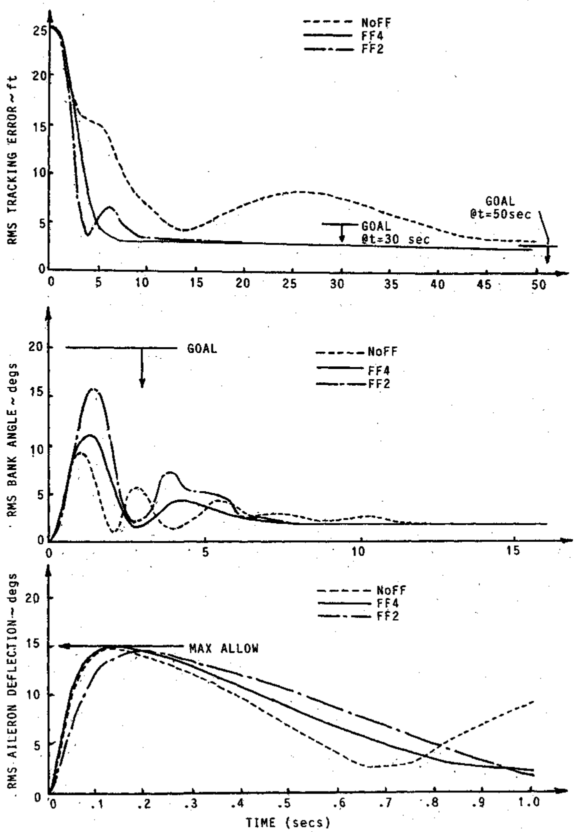

Fig. 7 RMS time histories for RPV net capture example.

the goal is lowered to a maximum of 2.75 ft (i.e., the performance achieved with the modified QPI). If ρ is decreased until the conventional QPI meets this goal, a prohibitively large control response (> 45 deg) is seen to occur. Hence, design with the modified QPI is a better means for meeting a given error goal without violating the given control constraints.

Conclusions

The advantages of a design method based on the minimization of a quadratic performance index that allows the designer to weight transient and steady-state tracking responses separately have been demonstrated. An example was shown of a controller for a "double-integrator" plant that meets steady-state tracking accuracy requirements with 40–75% less peak control effort or, alternatively, meets control constraints while providing 38–54% improved steady-state tracking accuracy than controllers designed with previous methods.

The method was applied to the synthesis of low-order tracking autopilot designs for the shipboard net recovery of a remotely piloted vehicle. It was shown that the optimization of a second-order control structure using target-state feedforward resulted in an autopilot design that successfully met all transient and steady-state tracking objectives. The designing of the same controller structure using the conventional parameter optimization performance index with equal transient and steady-state weightings resulted in a 9% loss of steady-state tracking accuracy while meeting the same constraints on control effort. It was shown that an excessive amount of control authority would be required to match the steady-state accuracy provided by the design based on the modified performance index.

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